Letters

Comments on "Analysis of Superconducting Microwave Structures: Application to Microstrip Lines"

Y. L. Chow, R. Faraji-Dana, and S. Safavi-Naeini

The above paper [1] suggests a strong relation between the axial current distribution J_z over the cross section of a superconducting microstripline and the accumulated charge density ρ_s on its boundary surface. The "two fluid model," $\vec{J} = \vec{J}_n + \vec{J}_s$, and the London's equation

$$\vec{J}_s = \frac{1}{j\omega\mu_0\lambda^2}\vec{E} \tag{1}$$

are used in the paper to model the superconducting materials. In this model, superconductors are treated macroscopically as ordinary conductors with complex conductivity $\tilde{\sigma} = \sigma_n - j\sigma_{sc}$ and $\sigma_{sc} = 1/\omega\mu_0\lambda^2$ (see [2] for instance); therefore, the claimed relation between J_z and ρ_s is supposed to be valid for normal conductors. In fact, in the previous paper [3] of the authors of [1], a similar proportionality relation between J_z and ρ_s is reported for microstriplines made of normal conductors. The obvious result of this proportionality relation for a microstripline with substrate of high dielectric constant, is a tremendous concentration of both charge and axial current distributions on the dielectric side (lower side) of the strip. In fact a concentration of the lower side current density of 30 to 60 times larger than the upper side of the strip is reported in these two papers for a dielectric constant of 23. Since this surprising result does not seem to comply with the known experimental observations and other analyses, the purpose of this *letter* is to reconcile these contradictions. We believe that in both papers an invalid extension of the quasi-TEM approximation from a perfectly conducting strip to the imperfect conductor strips created these incorrect conclusions.

I. Experimental Evidences

The effect of the substrate dielectric constant ϵ_r on the axial current density J_z of a normal strip conductor can be observed through the conductor loss measurements. In fact, high concentration of J_z on the lower side of the strip, reported in the above two papers, means a two fold increase in the attenuation constant of the line. This does not agree with the loss experiments reported by Pucel *et al.* [4] where no such increase was observed for increasing dielectric constant ϵ_r even for an ϵ_r of 105. Their measurements show good agreements with the conductor loss formula developed, which is based on the Wheeler's incremental inductance rule, which does not consider the existence of the dielectric substrate in the structure. Mittra and Itoh [5] have also mentioned this experimental verification. Recent published results [6] on GaAs microstrips loss measurements ($\epsilon_r = 12.9$) have also shown good agreement between experiments and loss calculations using existing software packages which ignore the existence of the dielectric substrate.

II. Analytical Evidences

In the paper, quasi-TEM approximations of the full wave governing equations ⟨16⟩[1] and ⟨18⟩ are used both inside and outside the superconducting strip. It is assumed that the current flows along the direction of the wave propagation, i.e., $\vec{J} = J_z\hat{z}$, and only the z-component of $\vec{A}$ exists, $\vec{A} = A_z\hat{z}$, and the governing equations become:

$$\left(\nabla_t^2 - \frac{1}{\lambda^2}\right)A_z = -\mu_0\nabla\psi\cdot\hat{z} \quad \text{inside the strip} \quad \langle 22\rangle \tag{2}$$

$$\nabla_t^2 A_z = 0 \quad \text{outside of the strip} \quad \langle 23\rangle \tag{3}$$

Obviously the most important part of this formulation is the choice of excitation function $\nabla\psi\cdot\hat{z}$ in (2). Equation ⟨20⟩ of the paper assumes a proportionality between $\nabla\psi\cdot\hat{z}$ and the charge distribution, that is,

$$\nabla\psi\cdot\hat{z} = \frac{\omega}{\beta}\rho_s = \nu_{ph}\rho_s. \quad \langle 20\rangle \tag{4}$$

Considering this highly nonuniform excitation over the upper and lower sides of the strip, the numerical output for the axial current distribution

$$J_z = \frac{-1}{\mu_0\lambda^2}A_z + \nabla\psi\cdot\hat{z} \quad \langle 15\rangle \tag{5}$$

should be very similar to the charge distribution, as reported in the paper. In this section we examine the validity of the assumptions which led to this unexpected result.

1) The assumption of a pure solenoidal vector potential $\vec{A}(\nabla\cdot\vec{A} = 0$ by Coulomb's gauge) requires more than one component of $\vec{A}$ in each region, for $\vec{A} = A_z\hat{z}$ gives

$$\nabla\cdot\vec{A} = 0 \Rightarrow -j\beta A_z = 0 \Rightarrow \beta = 0. \tag{6}$$

This *contradicts* the formulation, since $\beta = 0$ means that $\nabla\psi\cdot\hat{z} \equiv 0$ and the excitation problem (2) is converted to a homogeneous problem without the claimed dependence on the charge distribution. Therefore, the transverse components of $\vec{A}$ must also be included in the formulation especially in the interior region.

2) The paper uses a quasi-TEM approximation of the fields ($\partial/\partial x = \partial/\partial y = 0$) at the interior of the superconductor strip, to show the significance of $\nabla\psi$ in shaping the axial current distribution J_z. It then derives (4) as the demonstration of proportionality between J_z and ρ. At this equation, the paper states that the charge ρ is zero everywhere except near the surface. It was pointed out by Matthaei *et al.* [7] that the charge is actually a surface charge ρ_s. Using the full wave formulation of the paper one can easily derive from

$$\nabla\cdot\vec{J} = \nabla\cdot\left(\frac{\partial\psi}{\partial z}\hat{z} + \frac{\partial\psi}{\partial x}\hat{x} + \frac{\partial\psi}{\partial y}\hat{y}\right) = -j\omega\rho, \tag{6}$$

[1]The symbol ⟨ ⟩ is used in quoting the equation numbers of paper [1], to avoid confusion with the equation numbers of this letter.

Manuscript received April 17, 1992.
The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3G1.
IEEE Log Number 9202907.

the true dependence between $\vec{J}$ and ρ_s, that is, on the boundary,

$$\hat{n} \cdot \vec{J} = -j\omega\rho_s = \frac{\partial\psi}{\partial n}\Big|_{n-}^{n+}, \quad (7)$$

where $\hat{n}$ is the inward vector normal to the boundary of the strip. Equation (7) shows that:

a) As also pointed out by Matthaei *et al.* [7], the surface charge density on the boundary of a conductor is proportional to the normal component of the current density not the axial component. Therefore, the existence of a larger concentration of surface charge density on the lower side of the strip implies a larger concentration of the *normal* component of $\vec{J}$. However the total current I and charge per unit length q are always related through $I = (\omega/\beta)q$.

b) ρ_z is proportional to the normal component of $\nabla\psi$ and the claimed proportionality of $\nabla\psi \cdot \hat{z}$ with ρ_s is not correct. Also, assuming $\partial/\partial n = 0$, $n = x, y$ means no charge accumulation on the boundary.

As mentioned before, the above error arises from the invalid assumption $\partial/\partial x = \partial/\partial y = 0$ in ⟨19⟩ of the paper. In an exact sense, this equation should be written as

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - j\beta\nabla\psi \cdot \hat{z} = -j\omega\rho. \quad (8)$$

Therefore,

$$\nabla\psi \cdot \hat{z} = \frac{\omega}{\beta}\rho + \frac{1}{j\beta}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right). \quad (9)$$

Since the field distribution inside the conductor varies very rapidly in the transverse direction due to the skin effect, it is clear that $\partial/\partial x$ and $\partial/\partial y$ cannot be ignored.

3) Another clear contradiction arising from setting $\partial/\partial x = \partial/\partial y = 0$ in (9) is that $\psi(x, y)$ should be almost uniform with respect to x and y. This contradicts the claimed proportionality of ψ and ρ_s (⟨20⟩ of the paper) as clearly ρ_s is expected to have a highly nonuniform distribution on the lower and upper sides and also at the corners of the strip.

Based on the above facts, it is clear that for a wave propagation of $e^{-j\beta z}$, while a quasi-TEM approximation can be valid for the fields outside the strip [8] (exterior problem), it is not valid for the interior problem.

Finally, it should be mentioned that several published studies on the current distribution and conductor loss of a microstrip structure have achieved accurate results despite neglecting the presence of the dielectric substrate. In most of these studies a uniform excitation of the electric field have been assumed in the structure [9]–[13]. In our opinion, in the two papers being discussed here, lack of comparison with physically measurable parameters (like the conductor loss) has hidden the physical implications of the reported results on the current distribution. It is however expected that in paper [1], by implementing the correct excitation $\nabla\psi$ in the full wave formulation of the interior region and applying the correct boundary conditions as (7), the correct result of more even distribution of axial current densities on the upper and lower sides should be generated.

References

[1] S. M. El-Ghazaly, R. B. Hammond, and T. Itoh, "Analysis of superconducting microwave structures: Application to microstrip lines," *IEEE Trans. Microwave Theory Tech* , vol. 40, pp. 499–508, Mar. 1992.

[2] K. K. Mei and G. Liang, "Electromagnetics of superconductors," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1545–1552, Sept. 1991.

[3] S. M. El-Ghazaly, T. Itoh, and R. B. Hammond, "Design considerations for high T_c superconductor microstrip lines based on current distributions," *Microwave and Optical Tech. Lett.*, pp. 302–304, July 1991.

[4] R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Losses in Microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 342–350, June 1968.

[5] R. Mittra and T. Itoh, "Analysis of microstrip transmission lines," in *Advances in Microwaves*, L Young and H. Sobol, eds., vol. 8, New York: Academic Press, 1974, p. 126.

[6] M. E. Goldfarb, A. Platzker, "Losses in GaAs microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. 38, Dec. 1990, pp. 1957–1963.

[7] G. L. Matthaei, K. Kiziloglu, N. Dagli, and S. I. Long, "The nature of the charges, currents, and fields in and about conductors having cross sectional dimensions of the order of a skin depth," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1031–1036, Aug. 1990.

[8] A. C. Cangellaris, "A note on the calculation of the current distribution in lossy microstrip structures," *IEEE Microwave Guided Wave Lett.*, vol. 1, pp. 81–83, Apr. 1991.

[9] P. Waldow and I. Wolff, "The skin effect at high frequency," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1076–1082, Oct. 1985.

[10] A. R. Djordjevic, T. K. Sarkar, and S. M. Rao, "Analysis of finite conductivity cylindrical conductor excited by axially-independent TM electromagnetic field," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 960–966, Oct. 1985.

[11] P. Waldow and I. Wolff, "Dual bound variational formulation of skin effect problems," in *1987 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 333–336.

[12] A. C. Cangellaris, "The Importance of Skin Effect in Microstrip Lines at High Frequencies," in *MWSYM'88, Rec.* vol. 1, pp. 197–198.

[13] R. Faraji-Dana and Y. L. Chow, "The current distribution and ac resistance of a microstrip structure," *IEEE Trans. Microwave Theory Tech.*, vol. 38, Sept. 1990, pp. 1268–1277.

Reply to Comments on "Analysis of Superconducting Microwave Structures: Application to Microstrip Lines"

Samir M. El-Ghazaly

In the above mentioned letter [1], Chow *et al.* object to some observations presented in two of our papers [2]–[3]. Their objections can be summarized into the following topic points:

1. The distribution of longitudinal current component J_z in the transverse plane is related to the surface charge density ρ_s.
2. The distribution of J_z in the transverse plane varies with the dielectric constant of the substrate.

In their letter, they presented what they thought to be valid experimental and theoretical evidences. Unfortunately, they failed to understand some salient features of our papers. Also, they overestimate the applicability of Wheeler's approximation [4], namely the inductance per unit length is independent of the dielectric constant in transmission lines.

To clear the confusion generated by Chow *et al.* and to avoid indulging into massive numerical calculations that distract the at-

Manuscript received June 26, 1992.

The author is with the College of Engineering and Applied Sciences, Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287-5706.

IEEE Log Number 9202908.

tention away from the physics, this reply is organized into two sections. In Section I, Points 1 and 2 mentioned above will be discussed based on one of the simplest electromagnetic problems one can think of. In Section II, we will provide discussions to clear up the arguments presented by Chow *et al.* [1] as their experimental and analytical evidences.

Section I: Effect of Dielectric

To discuss Point 1, consider the parallel-plane transmission line presented in Fig. 1(a). The structure is lossless and fringing effects are neglected. A TEM wave propagating in the z-direction is excited. The electric field is directed in the x-direction and the magnetic field is in the y-direction. The commonly used symbols and terminology are used here. Using Maxwell's equations, the following relations can easily be derived.

$$\boldsymbol{H} = H_o e^{-j\beta z} \boldsymbol{a}_y, \tag{1}$$

and

$$\boldsymbol{E} = \eta H_o e^{-j\beta z} \boldsymbol{a}_x. \tag{2}$$

The surface charge density on the lower surface is

$$\rho_{sl} = \epsilon \boldsymbol{a}_x \cdot \boldsymbol{E} = \epsilon \eta H_o e^{-j\beta z}. \tag{3}$$

The surface current density on the lower surface is given by

$$\boldsymbol{J}_{sl} = \boldsymbol{a}_x x \boldsymbol{H} = H_o e^{-j\beta z} \boldsymbol{a}_z. \tag{4}$$

On the other hand, $\boldsymbol{J}_{sl}$ can alternatively be derived using the surface charge density and the phase velocity, $v_{ph} = 1/\sqrt{\mu\epsilon}$, as follows.

$$\boldsymbol{J}_{sl} = \rho_{sl} v_{ph} \boldsymbol{a}_z = (\epsilon \eta H_o e^{-j\beta z}) v_{ph} \boldsymbol{a}_z = H_o e^{-j\beta z} \boldsymbol{a}_z. \tag{5}$$

Obviously, (4) and (5) are identical, although the approach used in deriving them is different. Both of them prove the strong relation between the surface charge density and the axial current.

To discuss the effect of the dielectric constant on the current distribution, which is Point 2, consider the dielectric loaded parallel plate transmission line shown in Fig. 1(b). Using the symmetry of the structure, only one half is analyzed. Assume lossless dielectric material in the region $y < t$, and perfect conductors. Consider the lowest TE mode and using a magnetic-type Hertzian potential along the z-direction, it can be shown that the electromagnetic field expansion in the two regions are

For $y < t$

$$\boldsymbol{H} = (-\boldsymbol{a}_y j\beta \cos(k_y y)/k_y + \boldsymbol{a}_z \sin(k_y y)) H_{od} e^{-j\beta z}, \tag{6}$$

$$\boldsymbol{E} = -\boldsymbol{a}_x j\omega\mu H_{od} \cos(k_y y)/k_y e^{-j\beta z}, \tag{7}$$

and for $y > t$

$$\boldsymbol{H} = (-\boldsymbol{a}_y j\beta e^{-\alpha y}/\alpha + \boldsymbol{a}_z e^{-\alpha y}) H_{oa} e^{-j\beta z}, \tag{8}$$

$$\boldsymbol{E} = -\boldsymbol{a}_x j\omega\mu H_{oa} e^{-\alpha y}/\alpha e^{-j\beta z}, \tag{9}$$

where

$$\beta^2 = \omega^2 \mu \epsilon_o \epsilon_r - k_y^2 = \omega^2 \mu \epsilon_o + \alpha^2. \tag{10}$$

H_{od} and H_{oa} are amplitude constants, k_y and α are the propagation parameters in the y-direction in the dielectric and air regions respectively, and β is the propagation constant along the z-direction. Matching the tangential components at $y = t$ and using (10), it can be shown that

$$k_y \tan(k_y t) = \alpha. \tag{11}$$

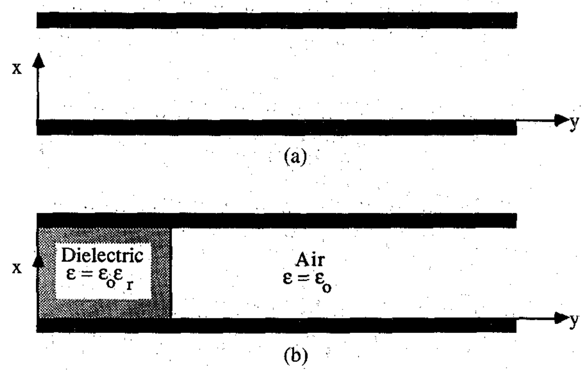

Fig. 1.

Using the y-component of the magnetic field, the surface current on the lower surface is given by

$$\begin{aligned} \boldsymbol{J}_{sl} &= (-\boldsymbol{a}_y \sin(k_y y) - \boldsymbol{a}_z j\beta \cos(k_y y)/k_y) H_{od} e^{-j\beta z}, \\ &\quad \text{for } y \le t \\ &= (-\boldsymbol{a}_y e^{-\alpha y} - \boldsymbol{a}_z j\beta e^{-\alpha y}/\alpha) H_{od} \sin(k_y t) e^{\alpha t} e^{-j\beta z}, \\ &\quad \text{for } y \ge t. \end{aligned} \tag{12}$$

Obviously, the current distributions given by (12) are distinctly different from those given by (4). This example decisively proves that the insertion of a dielectric material changes the surface current distributions. On other hand, when the space is uniformly filled with the same dielectric material, the current distribution is not changed. The magnitude of the current however changes, for the same external excitation, to reflect the change in the characteristic impedance of the line. Another electromagnetic fact, Chow *et al.* objected to, that can be demonstrated using this simple analysis is the current accumulation on surfaces adjacent to high dielectric constant regions. Using (12), the axial component of $\boldsymbol{J}_{sl}$ is integrated to evaluate the current carried by the surface adjacent to the dielectric I_d and the current carried by the conductor adjacent to the air region I_a as follows.

$$I_d = \int_0^t \boldsymbol{J}_{sl} \cdot \boldsymbol{a}_z \, dy = \frac{-j\beta}{k_y^2} H_{od} \sin(k_y t), \tag{13}$$

and

$$I_a = \int_t^\infty \boldsymbol{J}_{sl} \cdot \boldsymbol{a}_z \, dy = \frac{-j\beta}{\alpha^2} H_{od} \sin(k_y t). \tag{14}$$

Hence, the ratio of I_d to the total current becomes

$$\frac{I_d}{I_{\text{total}}} = \frac{1}{1 + \left(\frac{k_y}{\alpha}\right)^2}. \tag{15}$$

This ratio is plotted in Fig. 2 for relative dielectric constants ranging from 2 to 60. This figure is developed for a frequency of 1 GHz. The analysis is independent of the separation between the two electrodes as long as the attention is focused on the fundamental mode. Fig. 2 shows that up to 80% of the current is carried by a small region of the conductor at high dielectric constants. It is a well known fact that the energy density concentration increases in high dielectric constants areas. In principle, this energy concentration is what carried forward as a transmitted power in transmission lines. Therefore, it must be associated with a concentration of the axial current on the conductors since the axial current is what car-

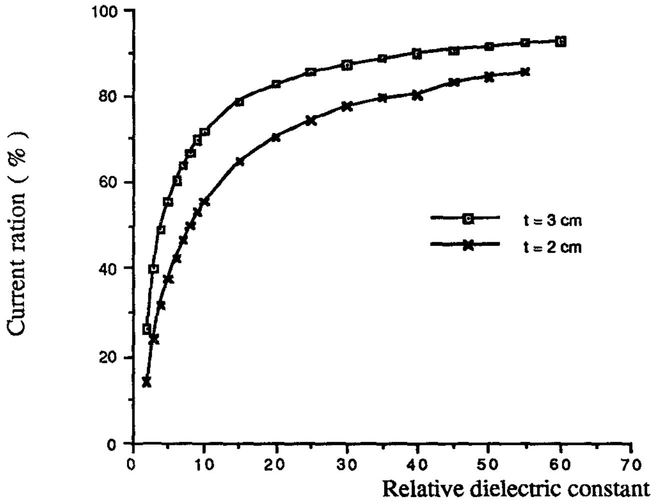

Fig. 2.

ries the power in the transmission line concept. The transmission line concept is in full agreement with Maxwell's equations. Actually, it is based on Maxwell's equations.

The last step in this section is to address the relation between transmission lines with perfect conductors and imperfect conductors. In perfect conductors, the current is concentrated on the surface; this is why it is called surface current density. As the conductivity is reduced from infinite to finite, the current spreads over a few skin depths. It is called current density in this case. Extending perfect conductor results to good conductors is simply achieved through the perturbation approach, which is quite acceptable for good conductors. Therefore, the above presented observation and points must also exist in good conductors and superconductors as well. The confusion presented in [1] can be cleared by reviewing the concept of surface impedance to clearly understand this point. Also, more discussions regarding extending perfect conductor results to good conductors are found in many text books (e.g., [6] Sec. 10-4 and [7] Section 7.04–7.06).

Section II: Reply to the Arguments

The discussion presented in Section I proves the physical observations presented in our papers [2] and [3]. To avoid creating any controversy, we will briefly point out to the main problems in their letter that lead to their incorrect conclusions. The following discussions show that our papers [2] and [3] are in full agreement with the electromagnetic fundamentals, and not contradicting the experimental evidences. Also, we will reconcile some physics pertaining to current distributions associated with guided waves.

A. Reconciling the Experimental Evidences

Chow *et al.* claim that experimental evidences support their views. They reference Paucel *et al.* [8], who made experiments using two different dielectric constants, 9.35 and 105. They also mentioned another experiment on GaAs. However, they drew the wrong conclusion from these experiments based on the following.

1. The experimental evidences presented by Chow *et al.* do not contradict our observations. The results by Pucel *et al.* can be understood by considering Fig. 4 in the same paper, [8]. This figure indicates that the current carried by the bottom surface is much larger than that carried by the upper surface. This observation is confirmed by Faraji-Dana and Chow in Fig. 5(b) in [9] (the first two authors of Chow *et al.* letter). In general, this observation is acceptable for the practical ranges of strip width to substrate thickness ratios. Since the conductor losses are proportional to the current square, then before tacking the dielectric substrate effects into consideration, the losses are mainly contributed by the lower surface of the conducting strip. Adding the substrate to this case increases the ratio of the current on the lower surface to the current on the upper surface, but does not practically increase the losses and the attenuation. To make it easy to follow this explanation, we will clear it up numerically using their Fig. 5(b) in [9]. The ratio between the two currents is *roughly* 10 to 1. For total current of 1 A, about 0.91 A flows on the lower surface. The losses from the upper surface are roughly about 1% of the total losses. Thus the losses are practically caused by the lower surface. Introducing a dielectric substrate in their structure increases this current ratio (say 100 to 1). The current on the lower surface becomes 0.99 A, for the same total drive current of 1 A. Consequently, the losses are still caused by the lower surface and no practical increase in the attenuation is observed. Fig. 3 in our paper [3] is very helpful in understanding this point. The increase in the current ratio should not be interpreted as increase in the current itself. Our results do not mean that the attenuation should increase with the substrate's dielectric constant.
2. Chow *et al.* mentioned only two uncorrelated results by two different authors, namely [8] and [10]. They fail to observe that the issue being discussed can only be decided by a systematic study, in which all parameters are kept unchanged and the dielectric constant is varied starting from dielectric constants close to one and up. The range of dielectric constant and the structure geometry (i.e., strip width and substrate thickness) are critical for observing the increase in attenuation. As it is explained in the previous paragraph, if one starts with a combination of a dielectric constant and a geometry which accumulates the current on one side of the strip, increasing the dielectric constant in the substrate will not lead to any observable increase in the losses in the conducting strip.

The above explanation demonstrates that our results are not contradicting the experimental *evidences* presented in [1].

B. Reconciling the Analytical Evidences

Chow *et al.* presented three points in this section. The response to them is as follows

1. The discussion presented by Chow *et al.* in their eq. (6) stems from misunderstanding of the simplified model presented in [2]. To obtain this model, we assumed $\xi \approx 1$, which means that $\omega \to 0$. Hence, when $\beta \to 0$, this does not imply that $\nabla \Psi \cdot \mathbf{a}_z$ should vanish. It should be noticed that $\omega/\beta = 0/0$ is an undefined quantity.
2. The response of this point has been completely discussed in Section I. The relations between the charge, the longitudinal current and the electric field are demonstrated clearly there. Chow *et al.* quoted correct information from Matthaei *et al.*, but unfortunately they drew the wrong conclusion from them. In the lossy case, the transverse electric field does penetrate the lossy conductor, where it terminates on electric charges distributed near the surface. That is what causes the normal components of the current. As the electromagnetic wave

propagates, the pattern of the transverse electric field moves along with it; actually the movement of the field patters is what we call a wave propagation. As the transverse electric field advances, it drags the charges inside the conductor with it, which leads to the axial current flow. Visualizing this physical picture saves researchers the trouble of interpreting mathematical expressions without a sense of direction, which may lead to wrong conclusions.

3. Chow *et al.* misunderstood and misquoted our paper regarding the assumption $\partial/\partial x = \partial/\partial y = 0$. They thought that we made this approximation for numerical calculations in our paper [2]. In [2], it is pointed out that this approximation was mentioned *only* to extract the physical meaning of $\nabla\Psi$. It was never used in the analysis nor in the simplified solution. Moreover, the assumption $\partial/\partial x = \partial/\partial y = 0$ is a special case, or represents a subcategory, of TEM waves. In general, TEM waves require the transverse divergence operator to be

$$\nabla_t = \partial/\partial x \boldsymbol{a}_x + \partial/\partial y \boldsymbol{a}_y = 0. \tag{16}$$

Thus TEM, or quasi TEM, waves allow variations in the x- and y-directions. A simple example of a TEM wave which is not a uniform plane wave is the fundamental mode of the microstrip line when the entire space is uniformly filled with the same dielectric material (e.g., when the substrate is removed to obtain an air-filled microstrip line). This structure supports a pure TEM-wave, while allowing both the electric and magnetic fields to vary in the transverse plane. Also the charge and current singularities are strongly pronounced despite the fact that this is a TEM-wave.

Finally, we would like to point out that the authors who utilized the incremental inductance approximation, including Wheeler himself, Pucel *et al.* and Mittra and Itoh, all of them acknowledged that this is just an approximation valid only under certain conditions. Therefore, when a researcher presents results that may not agree with the incremental inductance rule, nobody should automatically assume that the new results are wrong.

It is planned to execute a more accurate analysis based on the full wave solution presented in [2]. The accuracy of the numerical results presented in [2] will be checked against results of the more rigorous analysis.

In conclusion, we affirm that the axial current distributions in superconducting microstrip lines has a strong dependence on the charge density and the substrate's dielectric constant. Also, this strong dependence on the dielectric constant does not automatically lead to a measurable increase in the attenuation.

References

[1] Y. L. Chow, R. Faraji-Dana, and S. Safavi-Naeini, "Comments," *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 9, Sept. 1992.

[2] S. M. El-Ghazaly, R. B. Hammond, and T. Itoh, *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 499–508, 1992.

[3] ——, *Microwave and Optical Tech. Lett.*, pp. 302–304, July 1991.

[4] H. A. Wheeler, *IEEE Trans. Microwave Theory Tech.*, vol. 13, pp. 172–185, Mar. 1965.

[5] D. K. Cheng, *Field and Wave Electromagnetics*. Reading, MA: Addison-Wesley, 2nd ed., 1989.

[6] P. C. Magnusson, G. C. Alexander, and V. K. Tripathi, *Transmission Lines and Wave Propagation*. Boca Raton, FL: CRC Press, 3rd ed., 1992.

[7] S. Ramo, J. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1st ed., 1965.

[8] R. A. Pucel, D. J. Massé, and C. P. Hartwing, *IEEE Trans. Microwave Theory Tech.*, vol. 16, pp. 342–350, 1968.

[9] R. Faraji-Dana and Y. L. Chow, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1268–1277, Sept. 1990.

[10] G. L. Goldfarb and A. Platzker, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1957–1963, 1990.

Corrections to "A Study of the Nonorthogonal FDTD Method Versus the Conventional FDTD Technique for Computing Resonant Frequencies of Cylindrical Cavities

Paul H. Harms, Jin-Fa Lee, and Raj Mittra

A few corrections should be noted in our paper [1]. The reference numbers in Table I and II are incorrect, and a third reference should be added. Also, a mode shown for the results of the nonorthogonal FDTD analysis in Fig. 9 (b) was not included in Table II. This error results from the difficulty of identifying modes with the FDTD analysis, because only the frequency spectrum of the field is provided at a few points in the cavity. In comparing the FDTD results with data from the technical literature, it is difficult to match values mode-for-mode unless one knows what modes are

TABLE I
COMPARISON OF THE LOWER ORDER RESONANT FREQUENCIES FOR THE CYLINDRICAL CAVITY WITH A DIELECTRIC ROD FILLING ($\epsilon_r = 37.6$, $a = 1.00076$ cm, $b = 1.27$ cm, $L = 1.397$ cm)

Mode	Ref. [13] (GHz)	FEM [12] (GHz)	Nonorthogonal FDTD (GHz)
TM010	—	1.50	1.47
TM110	—	2.44	2.38
HE111	2.49	2.50	2.48
TM011	3.38	3.38	3.38
HE211	3.40	3.38	3.38
HE121	3.81	3.83	3.79

TABLE II
COMPARISON OF THE LOWER ORDER RESONANT FREQUENCIES FOR THE CYLINDRICAL CAVITY WITH A DIELECTRIC DISK FILLING ($\epsilon_r = 35.74$, $a = 0.8636$ cm, $b = 1.295$ cm, $H = 0.762$ cm, $L1 = L2 = 0.381$ cm)

Mode	Refs. [13], [15] (GHz)	FEM [12] (GHz)	Nonorthogonal FDTD (GHz)	% Difference (Ref. [13] & FDTD)
TE01	3.428	3.51	3.53	3.0%
HE11	4.224	4.27	3.90	7.7%
HE12	4.326	4.36	4.17	3.6%
TM01	4.551	4.54	4.53	0.5%

Manuscript received June 1.

J.-F. Lee was with the Electromagnetic Communication Laboratory, Department of Electrical and Computer Engineering, University of Illinois. He is presently with the Department of Electrical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609.

P. H. Harms and R. Mittra are with the Electromagnetic Communication Laboratory, Department of Electrical and Computer Engineering, University of Illinois, 1406 W. Green St., Urbana, IL 61801-2991.

IEEE Log Number 9202909.